1. 휴리스틱인지 휴리스틱스인지
2. 나이트 투어 설명
   1. 히스토리
      1. 언제부터 누가 시작했는지
      2. 근대에는 요즘은 어떻게 이야기하고 있는지
      3. 위키피디아 가져와도 됨
      4. 이야기의 스토리가 중요
3. 높은 차우에서의 결과의 증명을 보여주자
4. 휴리스틱이 뭔지 알려줘야 함
5. 조건에 관련도니 거는 GPT한테 물어보지
6. 개조식으로
7. 필요성에서는
   1. 기술의 단점 뭐가 문제인지
   2. 저차워에서는 문제가 되어도..
8. 컴퓨터 사양 적어야 됨
9. 다차원을 어필해야 함
10. Modified의 방식이 어떻게 돌아가는지 명확하게 이야기를 해야 한다.
11. 차이 명확하게 설명
12. 단점도 설명해야 돼
13. Time도 보여줘야 돼
14. W에서 거리가 작은 거 해 봐

A knight’s tour on an n × m chessboard is a traversal of the squares of a chessboard using only moves of the knight to visit each square once. A knight’s tour is closed if the last move of the tour returns the knight to its starting position; otherwise the tour is open.

In graph theoretical terms we can consider an n ×m chessboard as a grid of n ×m points. We associate with this grid a graph, the knight’s graph K(n,m), where each point is joined to all points a knight’s move away. Equivalently K(n,m) is the graph where V (G) = {(i, j) : 0 ≤ i ≤ n−1 , 0 ≤ j ≤ m−1} and {(i, j), (k, l)} ∈ E(G) ⇔ (i−k, j−l) ∈ {(±1,±2) , (±2,±1)}. So an n×m tour is precisely a Hamiltoniancycle in K(n,m).

“The oldest of knight puzzles is the knight‟s tour,” asserts Martin Gardner [1]. The problem is more than 1000 years old. The chess historian H. J. R. Murray [2] describes closed tours of the 8×8 board by the Shatranj players al-Adli and as-Suli who lived in Baghdad around 840 and 900 CE respectively. Knight's tour questions have continued to fascinate both amateur and professional mathematicians ever since. The mathematician A.T. Vandermonde [3] was the first to construct a three-dimensional knight's tour, in a 4 × 4 × 4 cube, published in 1771. Other 3D examples have been provided by Schubert [4], Gibbins [5], Stewart [6], Jelliss [7], Petkovic [8] and DeMaio [9].

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3. A. T. Vandermonde; Remarques sur les Problemes de Situation, Memoires de l'Academie des Sciences 1771.

4. H. Schubert, Mathematische Mussestunden Eine Sammlung von Geduldspielen, Kunststücken und Unterhaltungs-aufgaben mathematischer Natur. (Leipzig), 1904, p.235-7 4×4×4 closed tours, p.238 3×4×6 closed tour.

5. N. M. Gibbins, Chess in 3 and 4 dimensions, Mathematical Gazette, May 1944, pp 46-50.

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9. J. DeMaio, Which Chessboards have a Closed Knight‟s Tour within the Cube? The Electronic Journal of Combinatorics, 14, (2007) R32.

The knight's tour problem (KTP) is an interesting mathematical problem, and its history can be dated back to Euler and De Moivre [2]. In the past decade, the KTP has received considerable interest [11,8,5,6,9,7]. Recently, Chia and Ong [3] initiated

the study of the so-called generalized knight's tour problem (GKTP) by considering the more general .a; b/ move rather than

the (1, 2) move. In [10,4], the KTP was generalized to the 3D situation, still with (1, 2) move.

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2. WARNSDORFF’S RULE

In 1823, Warnsdorff [9] proposed a simple heuristic for finding knight’s tours.

Warnsdorff’s Rule: Always move to an adjacent, unvisited square with minimal degree.

Intuitively this seems like a logical rule to follow, since squares with lower degrees have fewer neighbors and

therefore we will have fewer opportunities to visit them in the remainder of the path. It is essential to follow

this rule if a square has degree 0, since otherwise we can never visit it. Similarly if we fail to visit a square of

degree 1, then we will have only one more opportunity to visit it (and it must be the last square of the tour). It

is also true that no tour can deviate from Warnsdorff’s Rule in the last several moves, although this is less

obvious. The idea here is that if we deviate at an early move we have a greater opportunity to overcome this

“mistake,” while if we deviate near the end of the path then we have fewer chances to avoid failure. Since this

“endgame effect” is central to the intuitive appeal of the heuristic, we will make it more rigorous with the

following theorem, which relies on a lemma.

In a 1996 REU paper Squirrel [8] performed 100 trials of this algorithm for all board sizes from 5 to 400; the

conclusion was that the algorithm is quite successful for smaller boards, but the success rate drops sharply as

board size increases. The algorithm produces a successful tour over 85% of the time on most boards with m

less than 50, and it succeeds over 50% of the time on most boards with m less than 100. However, for m > 200

the success rate is less than 5%, and for m > 325 there were no successes at all. These observations suggest

that the success rate of Warnsdorff’s random algorithm rapidly goes to 0 as m increases. I confirmed Squirrel’s

findings, and made another interesting observation. For all boards with m ≤ 25 the success rate was at least

98% with one surprising exception: for m = 7 the success rate was only 75%. I will have more to say about

this later in this section.

Despite the failure ofWarnsdorff’s proposed algorithm, it was believed thatWarnsdorff’s heuristic could still

be salvaged if appropriate improvementswere made. Several attempts have been made to improveWarnsdorff’s

algorithm; however, none of them proved successful on all boards. Parberry [4] considered an algorithm that

combined Warnsdorff’s heuristic with a random walk algorithm originally proposed by Euler in 1759. Euler’s

approach was to start at any square, then repeatedly move to a random unvisited neighbor until no more moves

are possible. Euler then attempted to replace moves of this path with longer sequences of moves consisting

of squares that were not visited by the path. A more complete analysis of Euler’s algorithm can be found in

[1]. Parberry decided to improve this algorithm by choosing the next square according toWarnsdorff’s random

algorithm; that is, chose a random square with minimal degree. While an interesting idea, experiments showed

that the success rate of the algorithm appears to decrease exponentially and the average running time required

to find each tour appears to increase exponentially.

Roth [6] decided that the problem lay in Warnsdorff’s random tiebreak rule. He proposed breaking ties by

choosing “the successor with largest euclidean distance to the center of the chessboard.” It was not comletely

clear how he would break ties if more than one square shared the same distance from the center of the board.

Roth claimed that his algorithm first failed on a board with 428 rows, and failed less than 1% of the time on all

boards with fewer than 2000 rows.

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